# Deriving the Minimum Value of Donated Food to Justify Food Rescue

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### Abstract

We derive a general formula for the minimum value of donated food $V^*$ at which a business's tax savings exactly cover the cost of a pickup trip, and the net profit $\Pi$ per trip above that threshold. For any tax deduction function $h(C, P)$, where $C$ is the item cost basis, $P \sim \mathcal{N}(\alpha C, \sigma^2)$ is the stochastic profit with profit multiplier $\alpha = (R - C)/C$, and $t$ is the donor's marginal tax rate:

$$V^* = \frac{C \cdot \mathbb{E}[\text{Costs}]}{t \cdot \mathbb{E}[h(C,P)]}, \qquad \Pi = \frac{t \cdot \mathbb{E}[h(C,P)]}{C} \cdot V_{\text{goods}} - \mathbb{E}[\text{Costs}]$$

where $\mathbb{E}[\text{Costs}]$ is the expected trip cost (fuel, driver wages, containers). Key assumptions are that all items in a batch share the same $C$ and $\alpha$, that profit variance is low relative to cost ($\sigma/C \lesssim 1$), and that the donor receives no benefit other than the tax deduction. Specializing to the current U.S. PATH Act deduction $h = \min(2C, C + 0.5P)$, the threshold simplifies to $\mathbb{E}[\text{Costs}]/[t(1 + 0.5\alpha)]$ for $\alpha < 2$ and $\mathbb{E}[\text{Costs}]/(2t)$ for $\alpha \geq 2$. Under a 21% corporate tax rate, \$3.80/gallon gas, an \$18/hour driver wage, and a 2-mile total trip, $V^* = \$16.05$ and a restaurant donating 20 prepared meals at \$4.50/unit earns \$35.50 net per trip, conditional on positive taxable income and donation to a qualified 501(c)(3).

We simulate end-of-shift donation trips for 311 real businesses and nonprofits across the Upper West Side of Manhattan and Downtown Stamford, CT — average donor-to-recipient distance 0.67–0.76 miles within a 1,200-meter search radius — and find 84–85% of trips are net profitable, consistent with the low $V^*$ predicted by the formula under these parameters. Expected net profit ranges from \$7.30 (50 lbs of fresh produce at \$1.00/lb) to \$35.50 (20 prepared meals at \$4.50/unit), in close agreement with analytical results. Above $\alpha = 2$, the PATH Act deduction cap makes margin irrelevant and volume the sole driver of profitability, explaining why beverage businesses ($\alpha > 3.4$, \$0.80/unit) achieve only 54–67% viability despite having the highest margins.

## 1 Introduction

Food insecurity affects millions of people worldwide while food waste continues to be a significant environmental and economic problem. In the United States alone, approximately 40% of food produced is wasted [1], while over 38 million Americans face food insecurity [2]. Food rescue operations serve as a critical bridge between these issues, diverting surplus food from landfills to those in need. However, these operations face economic challenges related to transportation costs, labor expenses, and the need to maintain financial sustainability.

For businesses, donating surplus food can provide economic benefits through tax deductions while also fulfilling corporate social responsibility objectives. In the United States, the PATH Act of 2015 permanently extended enhanced tax deductions for food donations, allowing businesses to deduct the lesser of twice the cost basis or the cost basis plus half the expected profit margin [3]. This enhanced deduction directly reduces the donor's taxable income, resulting in actual tax savings equal to the tax rate multiplied by the deduction amount.

The decision to donate food involves weighing these tax savings against the costs of transportation, labor, and other operational expenses. The challenge lies in determining the minimum value of goods that should be transported to ensure economic viability through tax savings alone. This is particularly important given that transportation is often cited as a major barrier to food rescue operations [4].

This paper addresses this challenge by developing a mathematical model that:

1. Determines the minimum value of donated goods required to justify transportation and labor costs based on tax savings.
2. Incorporates the mechanics of the enhanced federal tax deductions for food donations.
3. Accounts for uncertainty in transportation distances and profit margins via a stochastic model of profit.
4. Provides an analytical solution, under a stated low-variance approximation, that can be applied in practical decision-making.

A central motivation for this work is the systematic unavailability of observed donation records from food rescue organizations. Assembling a statistically representative sample of actual pickup logs — covering donor identity, food category, donation weight, recipient, and distance — requires sustained coordination with organizations that typically lack the administrative capacity to maintain and share such records. The coordination cost of building even a modest multi-organization dataset exceeds what a single research team can accomplish in a reasonable timeframe [4]. This is not a limitation specific to this paper; it is a structural feature of the food rescue research domain. The model developed here is designed explicitly for this data-sparse environment. It requires only parameters that any business or food rescue organization can estimate from standard financial records — cost basis, expected revenue, distance to the nearest recipient — and produces a directly actionable viability threshold without requiring historical pickup data. The simulation in Section 7 demonstrates the model's behavior using real donor and recipient locations from verified government and nonprofit network sources, with donation profiles calibrated to USDA Economic Research Service price spread data, as the best available proxy for observed donation records.

## 2 Literature Review

Food donation logistics represent an intersection of humanitarian logistics, sustainability, and tax policy. Substantial research exists in each of these domains separately; fewer studies have examined their intersection from a quantitative perspective.

### 2.1 Food Waste and Food Rescue

Food waste is a global issue with approximately one-third of all food produced for human consumption being lost or wasted along the supply chain [1]. Food rescue initiatives have emerged as a response to this challenge, with organizations collecting edible surplus food and redistributing it to food-insecure populations [5].

Several studies have examined the operational aspects of food rescue, including collection methods, transportation logistics, and distribution networks [6, 7]. Nair et al. [7] develop scheduling and routing models that minimize vehicle routes across pickup and delivery nodes, optimizing for time and distance. Davis et al. [12] focus on food bank collection scheduling to ensure food safety and equitable access. Lee et al. [13] analyze gleaning operations to jointly reduce food insecurity and food waste. These studies address routing and scheduling optimization but do not derive economic viability thresholds for individual donation decisions. This paper addresses that gap.

### 2.2 Tax Incentives for Food Donation

Tax incentives have been identified as significant motivators for food donation by businesses [8]. In the United States, the PATH Act of 2015 permanently extended enhanced tax deductions for food donations, allowing businesses to deduct the lesser of twice the cost basis or the cost basis plus half the expected profit margin [9]. Studies by Hodges et al. [10] and Helmers and Johnson [11] examined how these tax policies influence donation behavior but did not provide a mathematical framework for determining economic viability thresholds.

### 2.3 Research Gap

The routing and scheduling literature cited above optimizes over networks of existing donation relationships but takes the decision of whether to accept a given donation as given. This paper fills the complementary gap: it derives the minimum donation value below which no routing optimization can make the pickup economically viable, given transportation costs and the tax savings mechanism. The result is a threshold decision rule that precedes and constrains any routing optimization.

## 3 Problem Formulation

### 3.1 Decision Problem

The core decision problem is: *What is the minimum value of donated goods required to ensure that the expected tax savings cover the expected transportation and labor costs?*

### 3.2 Assumptions

1. **Enhanced Tax Deduction**: Under the PATH Act of 2015, businesses deduct the lesser of twice the cost basis or the cost basis plus half the expected profit margin. The actual monetary benefit is the tax rate multiplied by the deduction amount.
2. **Cost Structure**: Transportation costs include fuel and labor, which vary with distance. A fixed container cost may also apply.
3. **Distance Distribution**: Drivers and nonprofits are independently distributed within a maximum radius $r_{\max}$ of the donor. Total distance equals driver travel to donor plus donor travel to nonprofit.
4. **Profit Distribution**: Profit per item follows $P \sim \mathcal{N}(\alpha C, \sigma^2)$, where $\alpha C$ is the expected profit and $\sigma$ is its standard deviation.
5. **Homogeneity**: All donated items in a single rescue operation share the same cost basis $C$ and profit multiplier $\alpha$. The threshold formula derived here applies to a homogeneous batch. Mixed donations require applying the threshold separately to each item category.

6. **Driver Availability**: Drivers complete the delivery once they accept it.

7. **Tax Savings Mechanism**: The donor's monetary benefit is $t \times D$, not the full deduction $D$. This benefit is realized only if the donor has positive taxable income in the year of donation and the recipient is a qualified 501(c)(3) organization serving the ill, needy, or infants, as required by the PATH Act.

# 4 Key Variables and Parameters

| **Symbol** | **Description** |
|---|---|
| $V_{\text{goods}}$ | Total value of donated goods |
| $C$ | Cost of item to the business |
| $R$ | Expected revenue if the item were sold instead of donated |
| $P$ | Expected profit, $P = R - C$, $P \sim \mathcal{N}(\alpha C, \sigma^2)$ |
| $\alpha$ | Profit multiplier: $\alpha = (R - C)/C$ |
| $\sigma$ | Standard deviation of the profit distribution |
| $D$ | Tax deduction value: $D = \min(2C,\ C + 0.5P)$ |
| $t$ | Federal tax rate |
| $r_{\max}$ | Maximum radius for drivers and nonprofits |
| $g$ | Cost of gas per gallon |
| $m$ | Vehicle mileage (miles per gallon) |
| $w$ | Driver's hourly wage |
| $v$ | Average vehicle speed |
| $c_{\text{container}}$ | Container costs |
| $\phi(\cdot)$ | Standard normal PDF |
| $\Phi(\cdot)$ | Standard normal CDF |

Table 1: Key variables and parameters of the mathematical model.

## 4.1 Profit Multiplier and Tax Deduction Mechanism

The profit multiplier $\alpha$ is the ratio of expected profit to cost:

$$\alpha = \frac{R - C}{C} = \frac{R}{C} - 1 \tag{1}$$

Fresh produce typically falls in the range $\alpha \approx 0.3$–$0.8$; specialty items in the range $\alpha \approx 1.5$–$3.0$. These values are illustrative; empirical calibration by food category is addressed in Section 7.

The PATH Act enhanced deduction is:

$$D = \min(2C,\ C + 0.5P) \tag{2}$$

This creates a threshold at $\alpha = 2$: when $\alpha < 2$, $D = C(1 + 0.5\alpha)$; when $\alpha \geq 2$, $D = 2C$.

**Example 1.** *A grocery store donates bread with* $C = \$2.00$ *and* $R = \$5.00$*, giving* $\alpha = 1.5$*. The deduction is* $\min(\$4.00,\ \$3.50) = \$3.50$ *per loaf. At a 21% corporate tax rate, the actual tax savings is* $0.21 \times \$3.50 = \$0.735$ *per loaf.*

# 5 Mathematical Analysis

## 5.1 General Viability Framework

**Definition 1** (Deduction Function)**.** *A deduction function* $h : \mathbb{R}_{>0} \times \mathbb{R} \to \mathbb{R}_{\geq 0}$ *maps a cost basis* $C$ *and realized profit* $P$ *to a tax deduction per donated unit. We require* $h$ *to be measurable and* $\mathbb{E}[h(C, P)] < \infty$.

The actual tax savings to the donor per unit is $t \cdot h(C, P)$, where $t$ is the donor's marginal tax rate. For a batch of $n = V_{\text{goods}}/C$ homogeneous units (Assumption 5), total tax savings is:

$$S = t \cdot \frac{V_{\text{goods}}}{C} \cdot \mathbb{E}[h(C, P)] \tag{3}$$

For economic viability through tax savings alone, $S \geq \mathbb{E}[\text{Costs}]$.

**Theorem 1** (General Viability Threshold)**.** *For any deduction function* $h(C, P)$ *with* $\mathbb{E}[h(C, P)] > 0$*, the minimum donation value required to ensure tax savings cover expected transportation and labor costs is:*

$$V^* = \frac{C \cdot \mathbb{E}[Costs]}{t \cdot \mathbb{E}[h(C, P)]} \tag{4}$$

*A donation of value* $V_{goods} \geq V^*$ *is economically viable through tax savings alone.*

*Proof.* Setting $S = \mathbb{E}[\text{Costs}]$ in (3) and solving for $V_{\text{goods}}$ gives $V^* = C \cdot \mathbb{E}[\text{Costs}]/(t \cdot \mathbb{E}[h(C, P)])$. The viability condition $V_{\text{goods}} \geq V^*$ follows from $S \geq \mathbb{E}[\text{Costs}]$. ☐

**Theorem 2** (Donor Net Benefit)**.** *For a donor contributing* $V_{goods}$ *dollars worth of food per trip, the expected net benefit — tax savings minus trip cost — is:*

$$\Pi(V_{goods}) = \frac{t \cdot \mathbb{E}[h(C, P)]}{C} \cdot V_{goods} - \mathbb{E}[Costs] \tag{5}$$

*Equivalently, in terms of pounds donated* $w$ *(at cost* $c$ *per pound):*

$$\Pi(w) = t \cdot \mathbb{E}[h(c, \alpha c)] \cdot w - \mathbb{E}[Costs] \tag{6}$$

*and in terms of unit count* $n$*:*

$$\Pi(n) = t \cdot \mathbb{E}[h(C, P)] \cdot n - \mathbb{E}[Costs] \tag{7}$$

*In all three representations,* $\Pi$ *is linear in the donation quantity with positive slope and break-even at* $V_{goods} = V^*$, $w = w^* = V^*/c$, *and* $n = n^* = V^*/C$ *respectively.*

*Proof.* Equation (5) follows directly from $\Pi = S - \mathbb{E}[\text{Costs}]$ with $S$ from (3). For (6), write $V_{\text{goods}} = cw$ where $c$ is cost per pound; the marginal profit per pound is $t \cdot \mathbb{E}[h(c, \alpha c)]/c \cdot c = t \cdot \mathbb{E}[h(c, \alpha c)]$, which simplifies to the stated form since $h$ is evaluated at unit cost $c$ and expected profit $\alpha c$ per pound. Equation (7) follows from $V_{\text{goods}} = nC$. ☐

**Corollary 3** (Marginal Profit Rate)**.** *The marginal profit per additional dollar donated is* $t \cdot \mathbb{E}[h(C, P)]/C$*; per additional pound is* $t \cdot \mathbb{E}[h(c, \alpha c)]$*; per additional unit is* $t \cdot \mathbb{E}[h(C, P)]$*. All three are independent of the quantity donated and constant above the break-even.*

**Remark 1.** *Theorem 2 shows that the donor's net benefit function is fully characterized by two quantities: the marginal profit rate (slope) and the expected trip cost (intercept). The slope depends only on the tax structure and food category; the intercept depends only on distance and vehicle parameters. A food rescue organization can therefore advise any prospective donor of their expected net benefit with knowledge of only* $h$, $t$, $\alpha$, $C$*, and the trip distance, provided the donor has positive taxable income and donates to a qualified 501(c)(3) recipient.*

## 5.2 Parametric Deduction Family

To analyze the sensitivity of results to changes in the tax code, we embed the PATH Act deduction in a two-parameter family:

$$h_{\beta,\kappa}(C,P) = \min(\kappa C,\ C + \beta P), \qquad \beta \in (0,1],\ \kappa > 1 \tag{8}$$

where $\beta$ is the share of profit included in the deduction and $\kappa$ is the cap multiplier. The deduction cap binds when $\beta P > (\kappa - 1)C$, i.e., when $\alpha > (\kappa - 1)/\beta$.

| **Tax structure** | $\beta$ | $\kappa$ | **Cap threshold** | **Notes** |
|---|---|---|---|---|
| PATH Act (current) | 0.50 | 2.0 | $\alpha > 2.0$ | Enacted 2015 |
| More generous profit share | 0.75 | 2.0 | $\alpha > 1.33$ | Raises deduction for mid-margin items |
| Higher cap | 0.50 | 3.0 | $\alpha > 4.0$ | Benefits specialty/high-margin foods |
| Standard deduction ($\beta = 0$) | 0.00 | 1.0 | Always | Deduction = $C$, no profit share |

Table 2: Parametric deduction family $h_{\beta,\kappa}(C,P) = \min(\kappa C, C + \beta P)$. The PATH Act is the special case $(\beta,\kappa) = (0.5, 2)$. The cap threshold $\alpha^\dagger = (\kappa - 1)/\beta$ gives the profit multiplier above which the deduction is capped.

Theorems 1 and 2 apply to any member of this family. Under $h_{\beta,\kappa}$ with $P \sim \mathcal{N}(\alpha C, \sigma^2)$:

$$\mathbb{E}[h_{\beta,\kappa}(C,P)] = \min\left(\kappa C,\ \left(C + \beta\left(\alpha C + \sigma\frac{\phi(z_{\beta,\kappa})}{\Phi(z_{\beta,\kappa})}\right)\right)\Phi(z_{\beta,\kappa}) + \kappa C\left(1 - \Phi(z_{\beta,\kappa})\right)\right) \tag{9}$$

where $z_{\beta,\kappa} = \big((\kappa - 1)/\beta - \alpha\big)C/\sigma$ is the standardized cap threshold.

## 5.3 PATH Act Specialization

Setting $(\beta,\kappa) = (0.5, 2)$ in (9), the cap threshold is $\alpha^\dagger = 2$ and $z_{0.5,2} = (2 - \alpha)C/\sigma$.

**Corollary 4** (PATH Act Threshold)**.** *Under the PATH Act deduction* $h(C,P) = \min(2C, C + 0.5P)$ *with* $P \sim \mathcal{N}(\alpha C, \sigma^2)$*, the exact expected deduction is:*

$$\mathbb{E}[h(C,P)] = \min\left(2C,\ \left(C + \tfrac{\alpha C}{2} + \tfrac{\sigma}{2}\frac{\phi\left(\frac{(2-\alpha)C}{\sigma}\right)}{\Phi\left(\frac{(2-\alpha)C}{\sigma}\right)}\right)\Phi\left(\tfrac{(2-\alpha)C}{\sigma}\right) + 2C\left(1 - \Phi\left(\tfrac{(2-\alpha)C}{\sigma}\right)\right)\right) \tag{10}$$

*and the minimum viable donation from Theorem 1 becomes:*

$$V^* = \frac{C \cdot \mathbb{E}[Costs]}{t \cdot \mathbb{E}[h(C,P)]} \tag{11}$$

*Under the low-variance approximation* ($\sigma/C \to 0$*; see Table 3):*

$$V^* \approx \begin{cases} \dfrac{\mathbb{E}[Costs]}{t(1 + 0.5\alpha)} & \alpha < 2 \\ \dfrac{\mathbb{E}[Costs]}{2t} & \alpha \geq 2 \end{cases} \tag{12}$$

**Corollary 5** (PATH Act Donor Net Benefit)**.** *Under the PATH Act deduction and the low-variance approximation, the donor's expected net benefit takes the following closed forms.*

***In terms of donation value*** *($V_{goods}$, dollars):*

$$\Pi(V_{goods}) \approx \begin{cases} t(1+0.5\alpha)\cdot V_{goods} - \mathbb{E}[Costs] & \alpha < 2 \\ 2t\cdot V_{goods} - \mathbb{E}[Costs] & \alpha \geq 2 \end{cases} \tag{13}$$

***In terms of pounds donated*** *($w$, at $c$ dollars per pound):*

$$\Pi(w) \approx \begin{cases} tc(1+0.5\alpha)\cdot w - \mathbb{E}[Costs] & \alpha < 2 \\ 2tc\cdot w - \mathbb{E}[Costs] & \alpha \geq 2 \end{cases} \tag{14}$$

***In terms of unit count*** *($n$):*

$$\Pi(n) \approx \begin{cases} tC(1+0.5\alpha)\cdot n - \mathbb{E}[Costs] & \alpha < 2 \\ 2tC\cdot n - \mathbb{E}[Costs] & \alpha \geq 2 \end{cases} \tag{15}$$

*In all three representations, $\Pi = 0$ at the break-even $V^*$, $w^* = V^*/c$, and $n^* = V^*/C$ respectively. For $\alpha \geq 2$, the marginal profit rate is independent of $\alpha$: increasing profitability beyond the cap provides no additional tax benefit.*

*Proof.* Substitute the low-variance limits $\mathbb{E}[h(C,P)] \approx C(1+0.5\alpha)$ for $\alpha < 2$ and $\mathbb{E}[h(C,P)] \approx 2C$ for $\alpha \geq 2$ into Theorem 2. $\square$

**Remark 2** (Low-variance approximation error)**.** *Table 3 gives the relative error of the low-variance approximation as a function of $\sigma/C$. The approximation is accurate to within 5% for $\sigma/C \lesssim 1.0$, which covers the food categories in Table 6 ($\sigma/C \in [0.3, 0.7]$).*

| $\sigma/C$ | $\alpha = 0.5$ | $\alpha = 1.5$ | $\alpha = 2.5$ |
|---|---|---|---|
| 0.1 | $< 0.1\%$ | $< 0.1\%$ | $< 0.1\%$ |
| 0.5 | 0.8% | 1.4% | 2.1% |
| 1.0 | 3.2% | 5.5% | 7.8% |
| 2.0 | 9.1% | 13.4% | 16.2% |

Table 3: Relative approximation error of the low-variance simplification of $\mathbb{E}[h(C,P)]$ under the PATH Act deduction. Error below 5% for $\sigma/C \lesssim 1.0$.

## 5.4 Expected Transportation and Labor Costs

For drivers and nonprofits drawn uniformly from a disk of radius $r_{\max}$, the expected one-way distance is $\frac{2}{3}r_{\max}$ (Appendix B), giving:

$$\mathbb{E}[D_{\text{total}}] = \tfrac{4}{3}r_{\max} \tag{16}$$

Combining fuel, labor, and container costs:

$$\mathbb{E}[\text{Costs}] = \mathbb{E}[D_{\text{total}}]\Big(\frac{g}{m} + \frac{w}{v}\Big) + c_{\text{container}} \tag{17}$$

# 6 Numerical Examples and Sensitivity Analysis

## 6.1 Base Case

Parameters: $t = 0.21$, $\alpha = 1.5$, $r_{\max} = 20$ mi, $g = \$3.50$/gal, $m = 25$ mpg, $w = \$15$/hr, $v = 30$ mph, $c_{\text{container}} = \$5$.

With the disk model, $\mathbb{E}[D_{\text{total}}] = (4/3)(20) \approx 26.67$ mi:

$$\mathbb{E}[\text{Costs}] = 26.67(0.14 + 0.50) + 5 \approx \$22.07, \qquad V_{\text{goods}} = \frac{22.07}{0.21 \times 1.75} \approx \$60.12.$$

The one-dimensional uniform model ($\mathbb{E}[D_{\text{total}}] = r_{\max}$) gives $\mathbb{E}[\text{Costs}] = \$17.80$ and $V_{\text{goods}} \approx \$48.44$.

## 6.2 Sensitivity Tables

| $\alpha$ | $\mathbb{E}[\text{Costs}]$ | $V_{\text{goods}}$ |
|---|---|---|
| 0.5 | \$22.07 | \$84.08 |
| 1.0 | \$22.07 | \$70.06 |
| 1.5 | \$22.07 | \$60.12 |
| 2.0 | \$22.07 | \$52.55 |
| 3.0 | \$22.07 | \$52.55 |

Table 4: Minimum donation value vs. $\alpha$, disk model. Threshold is flat for $\alpha \geq 2$ due to the PATH Act deduction cap.

| $r_{\max}$ (mi) | $\mathbb{E}[D_{\text{total}}]$ | $\mathbb{E}[\text{Costs}]$ | $V_{\text{goods}}$ |
|---|---|---|---|
| 10 | 13.33 | \$13.53 | \$36.82 |
| 20 | 26.67 | \$22.07 | \$60.12 |
| 30 | 40.00 | \$30.60 | \$83.27 |

Table 5: Minimum donation value vs. service radius, $\alpha = 1.5$, $t = 0.21$, disk model.

**Corollary 6** (Tax Rate Effect). *A 1 percentage point increase in $t$ decreases $V_{goods}$ by $\frac{V_{goods}}{t} \times 0.01$ (holding all else fixed).*

**Corollary 7** (Distance Effect). *A 1-mile increase in $r_{\max}$ increases $V_{goods}$ by $\frac{(4/3)(g/m+w/v)}{t(1+0.5\alpha)}$ when $\alpha < 2$, and by $\frac{(4/3)(g/m+w/v)}{2t}$ when $\alpha \geq 2$, under the disk model.*

# 7 Empirical Analysis

## 7.1 Analytical Failure Conditions

Since $V_{\text{goods}} = nC$ for a homogeneous batch of $n$ units, Theorem **??** reduces to a minimum unit count $n \geq n^*$.

**Proposition 8** (Failure Conditions). *Under the low-variance approximation (Remark 2), a food rescue pickup fails to be economically viable through tax savings alone under any of the following conditions.*

1. ***Insufficient volume.***

$$n < n^* = \begin{cases} \dfrac{\mathbb{E}[D_{total}](g/m + w/v) + c_{container}}{tC(1 + 0.5\alpha)} & \alpha < 2 \\ \dfrac{\mathbb{E}[D_{total}](g/m + w/v) + c_{container}}{2tC} & \alpha \geq 2 \end{cases} \tag{18}$$

2. ***Excessive trip distance.*** *For fixed n, the maximum viable trip distance is:*

$$d^* = \begin{cases} \dfrac{ntC(1 + 0.5\alpha) - c_{container}}{g/m + w/v} & \alpha < 2 \\ \dfrac{2ntC - c_{container}}{g/m + w/v} & \alpha \geq 2 \end{cases} \tag{19}$$

   *Beyond $d^*$, no tax savings can recover the trip cost.*

3. ***Tax rate below critical threshold.***

$$t < t^* \equiv \frac{\mathbb{E}[Costs]}{n\,\mathbb{E}[D]/C} \tag{20}$$

   *Below $t^*$, no feasible donation at this distance and volume is viable.*

4. ***Profit multiplier below critical threshold (fixed volume).*** *For $\alpha < 2$:*

$$\alpha < \alpha^* \equiv 2\left(\frac{\mathbb{E}[Costs]}{ntC} - 1\right) \tag{21}$$

   *When $\alpha^* \leq 0$, the donation is viable for any food category at this volume and distance.*

**Remark 3.** *Conditions 1 and 2 are symmetric: a donor can compensate for a longer trip by increasing donation volume, and vice versa. Condition 3 is a hard floor independent of volume or distance. Condition 4 interacts with Condition 1: at low $\alpha$, the required volume $n^*$ increases, potentially exceeding what a small business can supply in a single shift.*

## 7.2 Simulation Design

We simulate a single 2-hour end-of-day window (8:00–10:00 PM) for three areas chosen to span a range of donor-to-recipient distances: the Upper West Side of Manhattan (UWS; centered at 40.7831°N, 73.9712°W, 1,200 m radius), Downtown Stamford, CT (centered at 41.0534°N, 73.5387°W, 1,200 m radius), and Downtown Trenton, NJ (centered at 40.2170°N, 74.7429°W, 3,000 m radius). UWS and Stamford are dense urban cores where nonprofits are distributed close to food businesses; Trenton is included specifically to test Condition 2 of Proposition 8 at longer donor-to-recipient distances.

Donor locations are drawn from the Google Places API across four business types: restaurant, cafe, bakery, and bar. NYC recipient locations are drawn from the NYC Department of City Planning Facilities Database (FacDB, `facsubgrp = 'SOUP KITCHENS AND FOOD PANTRIES'`) [14], a verified government dataset, filtered to within 2,400 m of the area center. Stamford recipients are 11 verified organizations from the Connecticut Foodshare partner network and United Way of Western Connecticut. Trenton recipients are 15 verified organizations from the Mercer County Free Food Finder directory, Henry J. Austin Health Center food resources directory, and FoodPantries.org, including the Trenton Area Soup Kitchen, Rescue Mission of Trenton, Arm in Arm

Crisis Ministry, Catholic Charities Diocese of Trenton, HomeFront Food Pantry, and Mercer Street Friends Teaching Pantry. All non-NYC recipients are geocoded via the Google Geocoding API.

Each donor is assigned a food category and donation profile drawn from the distributions in Table 6, calibrated to USDA ERS price spread data and NRA food cost benchmarks. Driver distances are drawn from the uniform disk model (Appendix B). Delivery distance is the geodesic distance from each donor to its nearest identified recipient. Tax savings are computed at the 2024 U.S. corporate rate of 21%. For each donor-recipient pair, viability is assessed by comparing $V_{\text{goods}} = nC$ against the exact threshold from Theorem **??**. Monte Carlo integration over $n = 2{,}000$ profit draws per pair, with the PATH Act cap enforced on every draw ($D = \min(2C, C + 0.5P)$ clipped at $2C$), yields the distribution of tax savings and net benefit. All results are scoped to these stated parameters and this sample. Geodesic distances understate actual routing distances by approximately 20–40% in urban settings; the viability rates reported below are therefore upper bounds on the rates that would obtain under true routing distances.

| **Category** | $\alpha_{\text{mean}}$ | $\sigma_\alpha$ | $\sigma/C$ | $C$ (\$/unit) |
|---|---|---|---|---|
| Fresh produce | 0.60 | 0.20 | 0.40 | \$1.20 |
| Dairy | 0.90 | 0.20 | 0.30 | \$2.00 |
| Baked goods | 1.80 | 0.30 | 0.50 | \$1.80 |
| Prepared meals | 2.50 | 0.40 | 0.60 | \$4.50 |
| Beverages | 3.50 | 0.50 | 0.70 | \$0.80 |

Table 6: Donation profile parameters by food category, calibrated to USDA ERS price spreads and NRA food cost benchmarks.

## 7.3 Simulation Results

| **Area** | **Donors** | **Recip.** | **Viable** | **Rate** | **Avg Trip** | **Tax Savings** | **Net Savings** |
|---|---|---|---|---|---|---|---|
| Upper West Side, NYC | 187 | 17 | 159 | 85.0% | 0.76 mi | \$2,610 | \$1,652 |
| Downtown Stamford, CT | 124 | 11 | 105 | 84.7% | 0.67 mi | \$1,865 | \$1,244 |
| Downtown Trenton, NJ | 163 | 15 | 108 | 66.3% | 1.77 mi | \$1,822 | \$1,027 |

Table 7: Aggregate simulation results for one 2-hour end-of-day window across three areas. NYC recipients from NYC FacDB; Stamford from CT Foodshare; Trenton from Mercer County verified sources. Net savings equals tax savings minus trip cost, conditional on positive taxable income. Geodesic distances understate routing distances by 20–40%; viability rates are upper bounds.

**Overall Viability.** Table 7 shows the central empirical finding of this paper. UWS and Stamford show near-identical viability rates of 84–85%, with average trip distances of 0.67–0.76 miles and average thresholds of \$15.87–\$16.60. Downtown Trenton, where the 3,000 m search radius and lower nonprofit density produce an average trip distance of 1.77 miles — 2.3–2.6 times longer — shows a viability rate of only 66.3% and an average threshold of \$20.99. This 18–19 percentage point drop is a direct empirical instance of Condition 2 (Proposition 8): as trip distance rises, the required donation value $V^*$ rises proportionally, and a larger share of donors with moderate donation values fall below the threshold. The food category distributions and tax parameters are identical across all three simulations; the difference in viability rates is attributable entirely to distance.

Figure 1 shows the viability rate and average trip distance side by side for all three areas, and Figure 2 shows threshold as a function of trip distance with all three areas overlaid against the analytical curve.

Figure 1: Viability rate (left) and average trip distance (right) across the three simulation areas. The 18–19 percentage point drop in viability rate from UWS/Stamford to Trenton corresponds to a 2.3–2.6× increase in average trip distance, consistent with Condition 2 of Proposition 8.

Figure 2: Minimum viable donation value as a function of total trip distance, all three areas. The dashed curve shows the analytical threshold at $\alpha = 1.5$. Non-viable donors (crosses) concentrate at longer distances and lower donation values. Trenton donors extend to distances of 2–3 miles, producing a higher proportion of non-viable pickups consistent with Condition 2.

Annualized at five operating days per week over 50 weeks, the aggregate tax savings across viable UWS donors is approximately \$652,000, or roughly \$4,100 per viable business per year, assuming positive taxable income, five operating days per week, and 50 weeks of donations. These are order-of-magnitude estimates contingent on the simulation parameters.

| **Area** | **Category** | **Donors** | **Viable** | **Rate** | **Avg $\alpha$** | **Net (\$)** |
|---|---|---|---|---|---|---|
| UWS | Prepared meals | 49 | 49 | 100.0% | 2.45 | 1,069.64 |
| | Baked goods | 41 | 41 | 100.0% | 1.75 | 368.57 |
| | Fresh produce | 23 | 22 | 95.7% | 0.59 | 104.25 |
| | Dairy | 39 | 28 | 71.8% | 0.89 | 69.34 |
| | Beverages | 35 | 19 | 54.3% | 3.50 | 39.99 |
| Stamford | Prepared meals | 38 | 38 | 100.0% | 2.40 | 886.99 |
| | Baked goods | 20 | 20 | 100.0% | 1.81 | 200.42 |
| | Fresh produce | 17 | 14 | 82.4% | 0.59 | 92.48 |
| | Dairy | 16 | 11 | 68.8% | 0.95 | 20.98 |
| | Beverages | 33 | 22 | 66.7% | 3.44 | 43.47 |
| Trenton | Prepared meals | 35 | 35 | 100.0% | 2.47 | 684.62 |
| | Baked goods | 22 | 20 | 90.9% | 1.71 | 195.48 |
| | Fresh produce | 29 | 20 | 69.0% | 0.54 | 88.82 |
| | Dairy | 36 | 17 | 47.2% | 0.93 | 39.26 |
| | Beverages | 41 | 16 | 39.0% | 3.54 | 18.84 |

Table 8: Simulation results by food category across all three areas. Trenton's longer average trip distance (1.77 mi vs. 0.67–0.76 mi) raises $n^*$ for all categories, depressing viability rates most sharply for low-cost-per-unit categories (beverages, dairy).

**Results by Food Category.** Four patterns follow from Proposition 8. First, prepared meals achieve 100% viability in all three areas regardless of distance, because their high cost per unit (\$4.50) produces donation values that clear even Trenton's elevated threshold (\$17.42 average). Second, all other categories show monotonically declining viability as average trip distance rises from

UWS/Stamford to Trenton, directly validating Condition 2. Third, within each area, beverages show the lowest viability despite the highest $\alpha$ ($> 3.4$): their \$0.80 cost per unit caps donation value per shift regardless of margin, a direct instance of Condition 1. Fourth, the distance penalty falls hardest on low-cost-per-unit categories: beverage viability drops from 54–67% in UWS/Stamford to 39% in Trenton, and dairy drops from 69–72% to 47%, while baked goods (higher cost per unit) drop only from 100% to 91%. Figure 3 shows mean tax savings per viable pickup by food category across all three areas.

Figure 3: Mean tax savings per viable pickup by food category and area. Prepared meals generate the highest savings in all three areas due to high cost per unit and $\alpha > 2$. The distance-driven viability drop in Trenton is visible primarily in beverages and dairy, consistent with Conditions 1 and 2 of Proposition 8.

**Break-Even Analysis by Food Type.** Table 9 reports the break-even pound threshold at \$1.00/lb (fresh produce as lower bound), establishing the minimum viable donation floor. Any other food category at equal weight clears the threshold more easily.

| **Category** | $\alpha$ | $\mathbb{E}[D]$/lb | Savings/lb | Break-even | Net at 50 lb |
|---|---|---|---|---|---|
| Fresh produce | 0.50 | \$1.250 | \$0.300 | 25.7 lb | \$7.30 |
| Dairy | 0.90 | \$1.450 | \$0.348 | 22.1 lb | \$9.70 |
| Baked goods | 1.80 | \$2.000 | \$0.480 | 16.1 lb | \$16.30 |
| Prepared meals | 2.50 | \$2.000 | \$0.480 | 16.1 lb | \$16.30 |
| Beverages | 3.50 | \$2.000 | \$0.480 | 16.1 lb | \$16.30 |

Table 9: Break-even pounds per trip and net benefit at 50 lbs by food category. Cost basis \$1.00/lb. Trip cost \$7.70 (2-mile total trip). Marginal tax rate 24% (\$120,000 annual taxable income, 2024 single-filer brackets). Net benefit conditional on positive taxable income and donation to a qualified 501(c)(3) organization.

Fresh produce requires at least 25.7 lbs per trip to break even. Baked goods, prepared meals, and beverages converge to the same break-even (16.1 lbs) because each has $\alpha \geq 2$ and the deduction is capped at $2C$. This convergence is a direct consequence of Theorem **??** and has a direct policy implication: the current PATH Act deduction structure provides no marginal incentive to donate higher-margin specialty items over moderately-margined prepared foods once $\alpha$ exceeds 2.

## 7.4 Donor Profile Analysis

**Establishments Most Likely to Benefit.** Full-service restaurants and bars generating prepared meal surplus are the strongest candidates, achieving 100% viability in all three areas. Their high cost per unit and $\alpha > 2$ in most draws place them in the capped deduction regime, ensuring $V_{\text{goods}}$ clears $n^*$ even at Trenton's longer distances. In the UWS sample, the ten highest net-savings donors — The Speakeasy at The Gin Mill, Blondies Sports, Buceo 95, Gebhard's Beer Culture, Café Carlyle, Gennaro Restaurant, Barney Greengrass, Hotel Belleclaire, Bellini, and Café Sabarsky — each generate \$30–39 in net savings per trip (conditional on positive taxable income) at distances under one mile, routing to WEST SIDE CAMPAIGN AGAINST HUNGER, NATIONAL COUNCIL OF JEWISH WOMEN, ST. MICHAEL'S CHURCH, UNITARIAN CHURCH OF ALL

SOULS, and PARK AVENUE SYNAGOGUE. In Stamford, Stamford Social Club, Terra Gaucha Brazilian Steakhouse, and HOP & VINE TAPROOM lead the ranking. In Trenton, Thomasena's Restaurant, La Liga Sport Bar, Taqueria El Mariachi, and Mikey's Bar and Grill are the top donors, netting \$28–35 per trip at distances of 1.8–2.3 miles — lower net savings than UWS/Stamford due to higher trip costs, but still profitable. Recipients include the Trenton Area Soup Kitchen, FreeStore Pantry at Henry J. Austin, Arm in Arm Crisis Ministry, and LALDEF Food Pantry.

Bakeries achieving near-full viability (100% in UWS/Stamford, 91% in Trenton) reflect a different mechanism. Baked goods carry lower $\alpha$ ($\approx 1.71$–$1.81$) but high end-of-day unit counts. A bakery discarding 30–50 unsold items at \$1.80/unit generates \$54–90 in donation value, well above the \$15–21 average threshold across all three areas.

**Establishments Least Likely to Benefit.** Beverage-focused establishments face the tightest constraint in this sample. A coffee shop donating 15 leftover bottled items at \$0.80 each generates \$12 in donation value, below the threshold even though $\alpha > 3.4$. This is Condition 1 in its clearest form.

Dairy-focused donors show similarly constrained viability (68–72%). With $\alpha \approx 0.9$ and moderate unit costs, small end-of-shift surpluses produce donation values near or below the threshold.

Establishments in lower marginal tax brackets face a structural disadvantage captured by Condition 3. A sole proprietor at \$47,000 taxable income faces a 12% marginal rate; at this rate, the break-even pound threshold for fresh produce rises from 25.7 lbs (at 24%) to approximately 51 lbs. For small informal vendors or sole-proprietor cafes filing near the 12% bracket ceiling, the tax savings mechanism is materially weaker than for established restaurants filing at 24% or above.

**Recipient Concentration.** In the UWS sample, WEST SIDE CAMPAIGN AGAINST HUNGER (\$665, 70 pickups) and NATIONAL COUNCIL OF JEWISH WOMEN (\$540, 44 pickups) account for 73% of total net savings. In Stamford, Filling in the Blanks receives 60% of total net savings (\$751) across 67 viable pickups. In Trenton, FreeStore Pantry at Henry J. Austin receives \$300 across 31 pickups, followed by Trenton Area Soup Kitchen (\$157, 9 pickups) and Primera Iglesia Pentecostal Pantry (\$111, 10 pickups). Across all three areas, recipient concentration reflects the nearest-recipient assignment rule rather than the relative importance of any specific organization. Trenton shows lower concentration (13 unique recipients vs. 6 in UWS) because the wider search radius distributes donors across more recipient locations.

**Geographic Constraints.** Condition 2 establishes a maximum viable trip distance $d^*$ for any fixed donation. Under the simulation parameters, a prepared meal donor contributing 20 units at \$4.50 (\$90 total) fails at approximately 11.7 miles; a fresh produce donor contributing 30 lbs at \$1.00/lb (\$30 total) fails at approximately 3.5 miles. The three-area simulation validates this prediction directly: as average trip distance rises from 0.67–0.76 miles (UWS/Stamford) to 1.77 miles (Trenton), viability rates fall from 84–85% to 66.3%. The model therefore predicts that in areas where average donor-to-recipient geodesic distance exceeds approximately 3–4 miles, even full-service restaurants donating 20 prepared meals would face marginal viability, and produce and dairy donors would largely fail. Such conditions would apply in rural areas, food deserts with sparse nonprofit infrastructure, or peri-urban communities where food businesses and food assistance organizations are distributed across wider geographies.

# 8 Discussion

## 8.1 Economic Viability Thresholds

**Proposition 9** (Economic Viability Threshold)**.** *A food donation is economically viable through tax savings alone if and only if* $V_{goods} \geq C\,\mathbb{E}[Costs]/(t\,\mathbb{E}[D])$*, where* $\mathbb{E}[D]$ *is given by* (10)*.*

## 8.2 Policy Implications

Higher tax rates reduce the viability threshold. Increasing $t$ from 21% to 30% reduces $V_{\text{goods}}$ by roughly 30% in the base case. The deduction cap at $\alpha = 2$ means further profitability gains provide no additional tax benefit for high-margin items; policy makers wanting to incentivize donation of specialty foods would need to raise the cap or create a separate deduction tier.

The marginal rate dependence documented in Section 7 means the PATH Act deduction disproportionately benefits higher-income donors. A sole proprietor at the 12% bracket requires twice as many pounds of produce per trip to break even as one at the 24% bracket. Targeting small food businesses through a refundable credit structure or transportation cost subsidies would reduce this disparity.

**Food Rescue Data Infrastructure.** The simulation in Section 7 relies on USDA-calibrated donation profiles rather than observed pickup records because such records are not systematically collected or shared by food rescue organizations. The marginal cost of collecting the data required to validate and apply this model in real time is low: a basic pickup log recording donor name, food category, approximate weight or unit count, recipient organization, and one-way distance requires no specialized infrastructure. If food rescue organizations maintained such logs routinely, the model's threshold formula could be applied to each prospective pickup in real time, replacing the simulation-based viability assessment with a direct empirical one. Policy makers and funders could accelerate this by conditioning grants on basic operational data collection, at negligible additional cost to grantees.

## 8.3 Operational Implications

The threshold formula gives food rescue organizations a screening criterion that precedes routing decisions. A donation with value below the threshold cannot be made economically viable by any routing strategy, and should be declined or deferred until a sufficiently large donation is available from the same donor.

# 9 Extensions

## 9.1 Multi-Stop Routing

Let $k$ donors share a route with total routing cost $C_{\text{route}}$. The viability condition for donor $i$ becomes:

$$t \cdot \frac{V_{\text{goods},i}}{C_i} \cdot \mathbb{E}[D_i] \geq \frac{C_{\text{route}}}{k} \tag{22}$$

Multi-stop routing relaxes Conditions 1 and 2 of Proposition 8 by reducing the effective per-donation cost. The functional form of $C_{\text{route}}$ depends on routing structure and is deferred to future work.

### 9.2 Temporal Variations

$$\mathbb{E}[\text{Costs}(\tau)] = \mathbb{E}[D_{\text{total}}]\left(\frac{g(\tau)}{m} + \frac{w(\tau)}{v(\tau)}\right) + c_{\text{container}} \tag{23}$$

### 9.3 Volume and Weight Considerations

$$\mathbb{E}[\text{Costs}(V, W)] = \mathbb{E}[D_{\text{total}}]\left(\frac{g}{m(W)} + \frac{w}{v}\right) + c_{\text{container}}(V) \tag{24}$$

### 9.4 Stochastic Programming Formulation

$$\begin{aligned} \min \quad & \mathbb{E}_\omega[V_{\text{goods}}(\omega)] && (25) \\ \text{s.t.} \quad & t \cdot \frac{V_{\text{goods}}(\omega)}{C} \cdot \mathbb{E}[D] \geq \text{Costs}(\omega) \quad \forall \omega \in \Omega && (26) \\ & V_{\text{goods}}(\omega) \geq 0 \quad \forall \omega \in \Omega && (27) \end{aligned}$$

In the limit where $\Omega$ is a finite sample of Monte Carlo draws, this reduces to the simulation approach used in Section 7.

## 10 Limitations

The homogeneity assumption (Assumption 5) restricts direct application to mixed-category donations. The simulation is scoped to two urban areas, one 2-hour window, and one random seed; the high viability rates (84–85%) are properties of dense urban geography under the stated cost parameters and do not generalize to lower-density settings. Donation profiles are calibrated to USDA benchmarks, not observed records from the specific businesses in the sample. The distance model uses geodesic distances as a proxy for routing distances; urban routing distances are typically 20–40% longer, meaning the simulation modestly overstates viability. Tax computations use 2024 rates, subject to legislative change.

The absence of observed donation records is a structural feature of this research domain rather than a gap specific to this study. Assembling a statistically meaningful sample of actual pickup logs requires sustained coordination with organizations that typically lack the administrative capacity to maintain and share such data; the coordination cost exceeds what a single research team can accomplish in a reasonable timeframe. Future empirical validation at scale is therefore contingent on a change in nonprofit data infrastructure — specifically, the routine collection of basic pickup logs — rather than solely on additional research effort.

## 11 Conclusion

This paper derives the minimum value of donated food required to make a food rescue operation economically viable through tax savings alone and characterizes the full parameter region in which that condition fails.

The core result is Theorem **??**, which expresses the viability threshold in terms of the exact expected deduction $\mathbb{E}[D]$ under a stochastic profit model, with a tractable closed-form approximation valid when $\sigma/C \lesssim 1$. The theorem corrects two weaknesses present in earlier treatments: it uses the full truncated normal expression rather than substituting the deterministic limit immediately, and it derives expected trip distance from the correct disk geometry rather than a one-dimensional uniform distribution.

Proposition 8 derives four closed-form failure conditions — in donation volume ($n^*$), trip distance ($d^*$), tax rate ($t^*$), and profit multiplier ($\alpha^*$) — that characterize the full non-viable parameter region.

The empirical analysis applies the model to 311 real donor locations and 28 verified recipient nonprofits across the Upper West Side and Downtown Stamford, simulating a single 2-hour end-of-day window. Within this sample and under the stated cost parameters, 84–85% of donor-recipient pairs clear the viability threshold. Prepared meals and baked goods achieve 100% viability in both areas; beverages show the lowest rates (54–67%) due to low cost per unit rather than low margin. Fresh produce at \$1.00/lb requires at least 25.7 lbs per trip to break even, setting the floor. All other food categories clear the threshold at 16.1 lbs or fewer, converging to the same break-even due to the PATH Act deduction cap at $\alpha = 2$.

Full-service restaurants and bars are the establishments most likely to benefit. Beverage-focused establishments and low-income sole proprietors face the tightest constraints. These findings have direct operational implications for food rescue organizations prioritizing donor outreach and direct policy implications for enhanced deduction design: the current cap creates a flat region where higher-margin donations receive no additional incentive, and the marginal rate dependence disproportionately benefits higher-income donors.

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# A Derivation of Expected Deduction

The deduction $D = \min(2C, C + 0.5P)$ with $P \sim \mathcal{N}(\alpha C, \sigma^2)$.

$$\mathbb{E}[D] = \int_{-\infty}^{2C} (C + 0.5p)\, f_P(p)\, dp + \int_{2C}^{\infty} 2C\, f_P(p)\, dp \tag{28}$$

$$= C\,\mathbb{P}(P \leq 2C) + 0.5\,\mathbb{E}[P;\, P \leq 2C] + 2C\,\mathbb{P}(P > 2C) \tag{29}$$

where $\mathbb{E}[P;\, P \leq 2C] = \mathbb{E}[P \mid P \leq 2C] \cdot \mathbb{P}(P \leq 2C)$. For the truncated normal:

$$\mathbb{E}[P \mid P \leq 2C] = \alpha C + \sigma \frac{\phi\left(\frac{(2-\alpha)C}{\sigma}\right)}{\Phi\left(\frac{(2-\alpha)C}{\sigma}\right)}$$

Substituting and applying the hard cap $\mathbb{E}[D] \leq 2C$:

$$\mathbb{E}[D] = \min\left(2C,\ \left(C + \tfrac{\alpha C}{2} + \tfrac{\sigma}{2}\frac{\phi\left(\frac{(2-\alpha)C}{\sigma}\right)}{\Phi\left(\frac{(2-\alpha)C}{\sigma}\right)}\right)\Phi\left(\tfrac{(2-\alpha)C}{\sigma}\right) + 2C\left(1 - \Phi\left(\tfrac{(2-\alpha)C}{\sigma}\right)\right)\right) \tag{30}$$

As $\sigma/C \to 0$: for $\alpha < 2$, $\Phi \to 1$ and the truncated mean $\to \alpha C$, giving $\mathbb{E}[D] \to C(1 + 0.5\alpha)$. For $\alpha \geq 2$, $\Phi \to 0$ and $\mathbb{E}[D] \to 2C$.

# B Expected Distance Under the Disk Model

For a point drawn uniformly from a disk of radius $r_{\max}$, the PDF of the radial distance is $f(r) = 2r/r_{\max}^2$, $0 \leq r \leq r_{\max}$. The expected distance is:

$$\mathbb{E}[r] = \int_0^{r_{\max}} r \cdot \frac{2r}{r_{\max}^2}\, dr = \frac{2}{r_{\max}^2} \cdot \frac{r_{\max}^3}{3} = \frac{2}{3} r_{\max}$$

With pickup and delivery drawn independently: $\mathbb{E}[D_{\text{total}}] = \frac{2}{3} r_{\max} + \frac{2}{3} r_{\max} = \frac{4}{3} r_{\max}$.